A Family of Mimetic Finite Difference Methods on Polygonal and Polyhedral Meshes

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In many applications, the mathematical model is formulated initially as a system of first-order partial differential equations, with each equation having a natural connection to physical aspects of the problem. For the diffusion problem these equations are

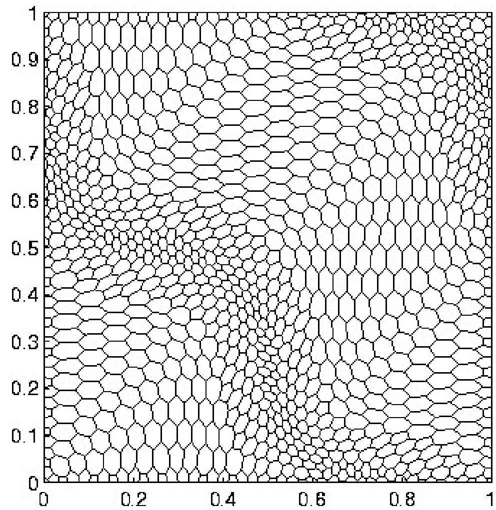
$$div\ F = b, \quad F = -K\ grad\ p,$$

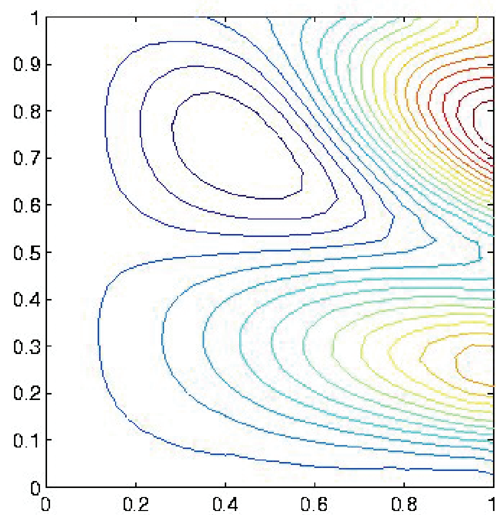
which describe the mass conservation and the Darcy law, respectively. The unknown variables are pressure *p* and flux *F*. The material properties are described by a full symmetric tensor *K*.

There are many discretization schemes with equivalent properties which can be used to solve the diffusion problem. The mimetic finite difference (MFD) method [1] is well suited for solving the first-order system, since it preserves essential properties (symmetry and mass conservation) of the continuum equations. The MFD method has been successfully employed for solving the diffusion problem on simplicial, quadrilateral, hexahedral, and unstructured polygonal and polyhedral meshes in both Cartesian and cylindrical coordinate systems.

In Ref. [2], we employed an innovative technique to give the first rigorous mathematical description of a rich family of MFD methods with equivalent properties. This family will allow us to tackle other computational problems such as enforcement of the discrete maximum principle.

We also developed a novel computationally inexpensive algorithm for deriving particular members of





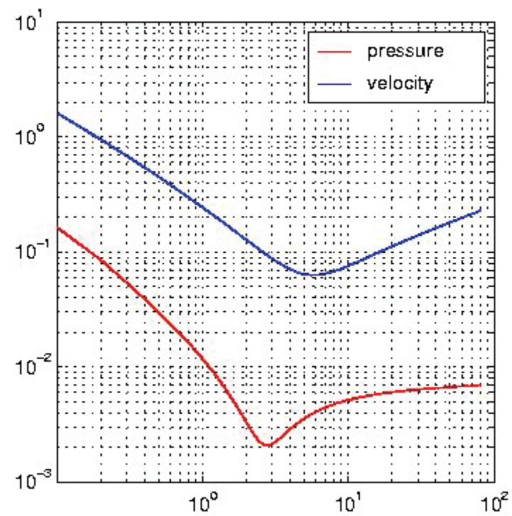


Fig. 1-3. ***The top picture shows polygonal mesh used in the convergence study. The middle picture shows the solution isolines. The bottom picture shows the mesh dependent L_2-norm of errors for pressure p and flux F (vertical axis) as functions of parameter u (horizontal axis).***

the family of MFD methods. These members are described by a single parameter. With this algorithm, solving the diffusion problems on a polyhedral mesh is as simple as on a tetrahedral mesh.

The illustrative example shows that there is a big interval for the parameter u where the discretization errors vary only 3 times. What is remarkable here is that for all values of u we observed second order convergence rate for the pressure and 1.5 convergence rate for the flux.

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[1] J. Hyman, et al., *Comput. Geosciences* **6**, 333–352 (2002).
[2] F. Brezzi, et al., *Math. Model. Methods Appl. Sci.* **15**, 10, 1533–1552 (2005).